2007 Paper 6 Question 12

Complexity Theory

Let **Bounded Factor** denote the following decision problem:

> Given positive integers n and k, decide whether n has a proper factor that is less than k.

For each of the following questions, give a detailed justification of your answer. Note that, in some cases, the answer may not be a simple "yes" or "no" but may instead be linked to open problems in complexity theory such as whether P=NP or the existence of one-way functions. In such cases, you should clearly explain the links and state what the consequences might be of both a positive and a negative answer to the question.

(a) Is **Bounded Factor** in NP?   [4 marks]

(b) Is **Bounded Factor** in co-NP?   [4 marks]

(c) Is **Bounded Factor** NP-hard?   [6 marks]

(d) Is **Bounded Factor** in P?   [6 marks]